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Beyond means: Distributional analysis of gender pay gaps

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Roadmap

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The gender wage gap increases with quantiles

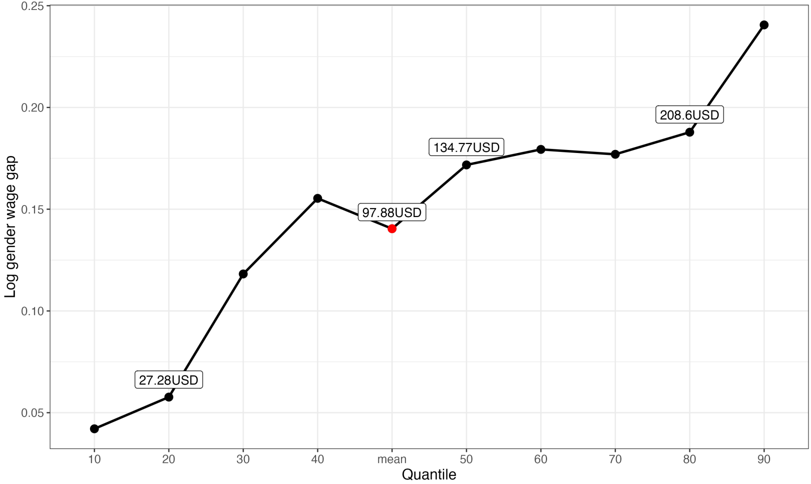

Figure: Distributional Gender Pay Gap in Chile (2013-2018).
Note: Gender pay gap is the difference in log earnings percentiles between males and females.

Related literature

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- Studies with a more “distributional" focus center on examining changes in the top salaries.
- The main explanation has to do with the impact of individuals' characteristics (Katz & Murphy, 1992).
- What is the role of firms? The gender wage inequalities literature has emphasized this. (Goldin, 2014; Goldin et al., 2017)
  1. Two-way FE (Abowd et al., 1999) $\Longrightarrow$ Card et al. (2016)
  2. Decomposition methods (Blinder, 1973; Oaxaca, 1973) $\Longrightarrow$ Blau and Kahn (2017)
- Primarily focus on **means**

Understanding (gender) wage gap (Card et al., 2016)

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Card et al., 2016 distinguishes between two approaches to address firm-specific pay policies that can be important for understanding the gender wage gap

(1) Sorting channel (between)

1. Women are less likely to find jobs at higher-paying firms
2. Wage gain for given firm-to-firm transition is smallest for women than men

(2) Bargaining channel (within)

- Wage-setting power of firms and the possibility that women are **offered or negotiate** systematically lower wages at a given firm

Basic framework

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Wage-setting (Abowd et al., 1999; Card et al., 2016)

$$\ln w_{it} = \underbrace{\alpha_i}_{\text{persons effect}} + \underbrace{\Psi^{G(i)}_{J(i,t)}}_{\text{gender firm effects}} + \underbrace{X'_{it}\beta^{G(i)}}_{\text{gender returns to the covariates}} + r_{it} \quad (1)$$

- $\psi^{G(i)}_{J(i,t)} \equiv \gamma^{G(i)} \bar{S}_{J(i,t)}$
- $r_{it} = \gamma^{G(i)}(\phi_{J(i,t)} + m_{J(i,t)}) + \varepsilon_{it}$

Decomposition (Blinder, 1973; Card et al., 2016; Oaxaca, 1973)

$$E[\Psi^{M}_{J(i,t)}|male] - E[\Psi^{F}_{J(i,t)}|female] \quad (2)$$

Decomposing the Effect of Firm-Level Wage Premiums

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$$E[\Psi^{M}_{J(i,t)}|male] - E[\Psi^{F}_{J(i,t)}|female] \quad (3)$$

1. Difference in mean values of the wage premium **within** the groups
2. Group difference in the wage premium (**between**)

Decomposing the Effect of Firm-Level Wage Premiums

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$$E[\Psi^M_{J(i,t)}|male] - E[\Psi^F_{J(i,t)}|female] + E[\Psi^F_{J(i,t)}|male] - E[\Psi^F_{J(i,t)}|male]$$

$$E[\Psi^F_{J(i,t)} - \Psi^M_{J(i,t)}|male] + E[\Psi^F_{J(i,t)}|male] - E[\Psi^F_{J(i,t)}|female] \quad (4)$$

Decomposing the Effect of Firm-Level Wage Premiums

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$$\underbrace{E[\Psi^{F}_{J(i,t)} - \Psi^{M}_{J(i,t)} | male]}_{\textbf{Bargaining power (within)}} + \underbrace{E[\Psi^{F}_{J(i,t)} | male] - E[\Psi^{F}_{J(i,t)} | female]}_{\textbf{Sorting effect (between)}}$$

1. **Average bargaining power** (*within firms*): Differential not explained by these difference observed in characteristic of women and men (across the distribution held by men)

Decomposing the Effect of Firm-Level Wage Premiums

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$$\underbrace{E[\Psi^F_{J(i,t)} - \Psi^M_{J(i,t)}|male]}_{\textbf{Bargaining power (within)}} + \underbrace{E[\Psi^F_{J(i,t)}|male] - E[\Psi^F_{J(i,t)}|female]}_{\textbf{Sorting effect (between)}}$$

1. **Average bargaining power** (*within firms*): Differential not explained by these difference observed in characteristic of women and men (across the distribution held by men)
2. **Average of sorting channel** (*between firms*): Differences in average wage premium across jobs held by men versus women

Distributional analysis (Firpo et al., 2009b)

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When the statistic of interest is a specific real-valued function of the outcome distribution,

$$\text{IF}\left(y;\nu\left(F_Y\right)\right)=\lim_{\varepsilon\to 0}\frac{\left[\nu\left(\left(1-\varepsilon\right)\cdot F_Y+\varepsilon\cdot\delta_y\right)-v\left(F_Y\right)\right]}{\varepsilon}=\frac{\partial\nu\left(\nu(F_Y)\right)}{\partial\varepsilon} \tag{5}$$

With $0\leq\varepsilon\leq 1$ and where:

- F signifies the cumulative distribution function of variable Y.
- δ_y is a distribution concentrated solely at the value y.

$$\begin{aligned}\text{RIF}\left(y;\nu(F_Y)\right)=\\ \nu\left(F_Y\right)+\int IF\left(y;\nu(F_Y)\right)\cdot dF_Y(y)\\ \nu\left(F_Y\right)+\text{IF}(y;\nu(F_Y))\end{aligned} \tag{6}$$

Distributional analysis (Firpo et al., 2009b)

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- The intuition: $F_Y(y) = \int F_{Y|X}(Y \mid X = x) dF_X(x)$

Property 1 - Integral relationship

$$\nu(F_Y) = \int RIF(y; \nu(F_Y)) dF_y(y) = E[RIF(y; \nu(F_Y))]$$

Theorem 1 - Integration for the Marginal Effect of a change in the distribution

$$\begin{aligned} \nu(F_Y) &= \int E\left(RIF\left(y; \nu(F_Y)\right) \mid X = x\right) dF_X(x) \\ &= E\{E\left(RIF\left(y; \nu(F_Y)\right) \mid X = x\right)\} \end{aligned}$$

Extending AKM to the case of quantiles

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- AKM version

$$\text{RIF}\left(y_i; q_\tau\right) = \alpha_{i\tau} + X_i'\beta_\tau + \Psi_{J(i,t),\tau} + \varepsilon_{i\tau}, \quad E\left(\varepsilon_{i\tau}\right) = 0$$

Extending AKM to the case of quantiles

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- AKM version

$$\text{RIF}\left(y_i; q_\tau\right) = \alpha_{i\tau} + X_i'\beta_\tau + \Psi_{J(i,t),\tau} + \varepsilon_{i\tau}, \quad E\left(\varepsilon_{i\tau}\right) = 0$$

- Property 1

$$q_\tau = E\left(RIF\left(y_i; q_\tau\right)\right) = E(\alpha_{i\tau}) + E\left(X_i'\beta_\tau\right) + E(\Psi_{J(i,t),\tau}) + E\left(\varepsilon_{i\tau}\right)$$

Extending AKM to the case of quantiles

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- AKM version

$$\text{RIF}\left(y_i; q_\tau\right) = \alpha_{i\tau} + X_i'\beta_\tau + \Psi_{J(i,t),\tau} + \varepsilon_{i\tau}, \quad E\left(\varepsilon_{i\tau}\right) = 0$$

- Property 1

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- Theorem 1 (LIE)

$$q_\tau = E\{E\left(RIF\left(y_i; q_\tau\right) \mid \cdot\right)\}$$

$$= E\{E(\alpha_{i\tau}) + E\left(X_i'\beta_\tau\right) + E(\Psi_{J(i,t),\tau}) + E\left(\varepsilon_{i\tau}\right) \mid \cdot\ \}$$

$$= E\{E(\alpha_{i\tau}) \mid \cdot\ \} + E\{E\left(X_i'\beta_\tau\right) \mid \cdot\ \} + E\{E(\Psi_{J(i,t),\tau}) \mid \cdot\ \}$$

With $E(\varepsilon_{i\tau}) = 0$ and $\cdot = \alpha_{i\tau}, X_i'\beta_\tau, \Psi_{J(i,t),\tau}$

Extending CCK Descomposition

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In general

$$\Delta_o^\nu = \nu\left(F_{Y_1|D_1}\right) - \nu\left(F_{Y_0|D_0}\right) = \underbrace{\nu\left(F_{Y_1|D_1}\right) - \nu\left(F_{Y_1|D_0}\right)}_{\Delta_S^\nu} + \underbrace{\nu\left(F_{Y_1|D_0}\right) - \nu\left(F_{Y_1|D_0}\right)}_{\Delta_X^\nu}$$

For firm premiums

$$\Delta_\Psi^\nu = \underbrace{\mathbb{E}[\Psi \mid D=1]'\left(\gamma_1^\nu - \gamma_0^\nu\right)}_{\Delta_S^\nu} + \underbrace{\left(\mathbb{E}[\Psi \mid D=1] - \mathbb{E}[\Psi \mid D=0]\right)'\gamma_0^\nu}_{\Delta_X^\nu} \quad (7)$$

1. Data

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- Matched employer-employee data from chilean Unemployment Insurance (UI) Registry
- 42,796,782 observations, representing a panel of individuals per month from January 2013 to December 2018 ($t = 60$)
- Monthly information of taxable wages
- Data solely covers the formal sector covered by labor code
- Limit sample to individuals whose earnings meet or exceed the minimum wage requirement for each respective period

2. Goodness of fit

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Table: Summary of Estimated TWFE Models by method for all sample, females, and males (2013-2018)

	Indicator	Mean	10	20	50	80	90
all	N	42,786,782	42,786,782	42,786,782	42,786,782	42,786,782	42,786,782
	Person FE (θ)	1,198,798	1,198,798	1,198,798	1,198,798	1,198,798	1,198,798
	Firm FE (ϕ)	80,417	80,417	80,417	80,417	80,417	80,417
	R^2 adjusted	0.88	0.49	0.60	0.72	0.76	0.77
	RMSE	0.22	0.34	0.39	0.46	0.65	0.85
F	N	16,542,868	16,542,868	16,542,868	16,542,868	16,542,868	16,542,868
	Person FE (θ)	486,794	486,794	486,794	486,794	486,794	486,794
	Firm FE (ϕ)	80,417	80,417	80,417	80,417	80,417	80,417
	R^2 adjusted	0.89	0.50	0.59	0.72	0.77	0.76
	RMSE	0.21	0.31	0.29	0.42	0.59	0.83
M	N	26,237,430	26,237,430	26,237,430	26,237,430	26,237,430	26,237,430
	Person FE (θ)	711,681	711,681	711,681	711,681	711,681	711,681
	Firm FE (ϕ)	80,417	80,417	80,417	80,417	80,417	80,417
	R^2 adjusted	0.88	0.50	0.61	0.71	0.76	0.77
	RMSE	0.23	0.34	0.44	0.46	0.66	0.79

3. OB decomposition

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Table: Oaxaca-Blinder decompositions for Firm FE (2013-2018)

Method	Gender Pay gap	Male premium	Female premium	Gender premium gap	Sorting	Bargaining
10	0.091	0.125	0.177	-0.052	0.0260	-0.0780
20	0.151	0.178	0.199	-0.021	0.0485	-0.0700
50	0.241	0.244	0.144	0.100	0.0720	0.0280
80	0.276	0.206	0.114	0.092	0.0575	0.0345
90	0.314	0.242	0.098	0.144	0.0610	0.0825

Note: sorting and bargaining are calculated using both counterfactuals

(1) What is the role of firms?

Firm premium benefits women for lower quantiles, while for higher quantiles and above the 50th percentile, it benefits men

3. OB decomposition

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(2) Lower quantiles

Gender pay gap is reduced due to bargaining. Nevertheless, sorting counteracts this effect, benefiting men

3. OB decomposition

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Note: sorting and bargaining are calculated using both counterfactuals

(3) Upper quantiles and median

Sorting becomes more important than bargaining for explaining gender pay gaps.Both components benefit men in terms of wages

4. Unionization?

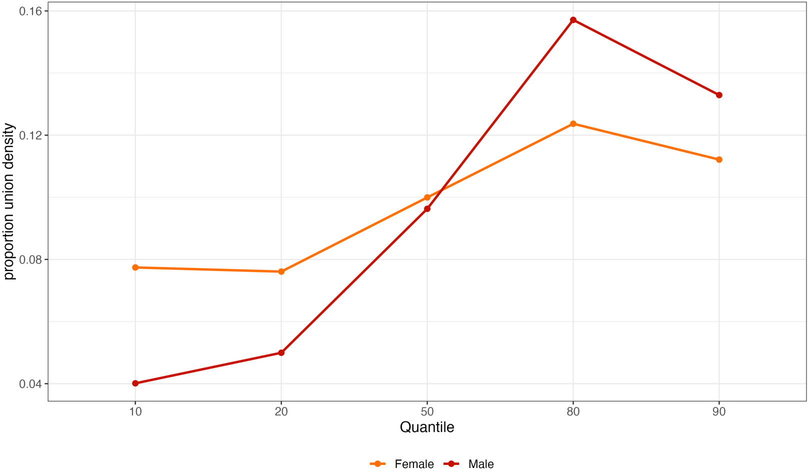

Figure: Union density by sex and quantiles of labor income.
Source: CASEN (2017) using private sector employees who meet the criteria of earning at least the minimum wage or more

5. Gender-differenciation in labor market

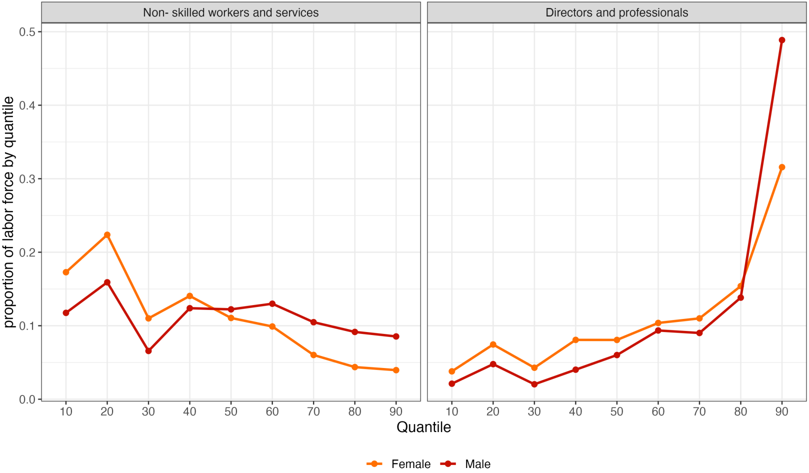

Figure: Labour force participation by sex and quantile.
Source: CASEN (2017) using private sector employees who meet the criteria of earning at least the minimum wage or more

Conclusion

1. Concerning the gender pay gap, we found that this gap widens as we move up the income quantiles.

Conclusion

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1. Concerning the gender pay gap, we found that this gap widens as we move up the income quantiles.
2. In the lowest quantile, firms helps reduce the gap, and in the highest quantile, firms contributes to its increase.

Conclusion

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1. Concerning the gender pay gap, we found that this gap widens as we move up the income quantiles.
2. In the lowest quantile, firms helps reduce the gap, and in the highest quantile, firms contributes to its increase.
3. We discovered that the bargaining dimension contributes to decrease the firm-driven gender pay gaps at the lower end of the income distribution, but not at the upper end.

Conclusion

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1. Concerning the gender pay gap, we found that this gap widens as we move up the income quantiles.
2. In the lowest quantile, firms helps reduce the gap, and in the highest quantile, firms contributes to its increase.
3. We discovered that the bargaining dimension contributes to decrease the firm-driven gender pay gaps at the lower end of the income distribution, but not at the upper end.
4. Next steps: Residual part? $\Longrightarrow$ Bargaining (Card et al., 2016) but we must to confirm unionization hypothesis.

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Thanks!

Beyond means: Distributional analysis of gender pay gaps

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Identification strategy (Card et al., 2018)

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- Identifying age and time effects: (1) difficult reconstructing a worker's employment history and (2) not clear employment gaps are exogenous.
- Normalization: choice of normalization is important for values of person-effects and time-varying controls.
- **Limited Mobility Bias**: (Bonhomme et al., 2019, 2020, 2023) parameters that are solely identified by workers who move across firms.
  - FE estimates of the **contribution of firm effects** to wage inequality are **upward biased**.
  - FE estimates of **contribution of the sorting of workers to firms are biased downward**.
- **Exogeneous Mobility**: worker and firm effect will be biased unless worker mobility is orthogonal to time-varying residual components of wages.
- Additive separability: it assumption about the proportional firm wage effect for all workers. (Card et al., 2013)
- **Distributions?**

Distributional Analysis (Firpo et al., 2009a)

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Property 1

$$\begin{aligned} \nu\left(F_Y\right) &= \int RIF\left(y;\nu,F_Y\right)dF_Y(y) \\ &= \int\int RIF\left(Y;\nu,F_Y\right)\cdot dF_{Y|X}(y \mid X=x)\cdot dF_X(x) \\ &= \int E\left[RIF\left(Y;\nu,F_Y\right) \mid X=x\right]\cdot dF_X(x) \end{aligned}$$

Distributional Analysis (Fortin et al., 2011)

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Equation 7

Let $Y = f(\Phi, \varepsilon) = \alpha_i + \Psi^{D_i}_{J(i,t)} + X'\beta^{D_i} + \varepsilon$

$$\begin{aligned}\Delta^{\nu}_{O} &= \mathbb{E}[Y \mid D = 1] - \mathbb{E}[Y \mid D = 0] \\ &= \mathbb{E}[\mathbb{E}(Y \mid \Phi, D = 1) \mid D = 1] - \mathbb{E}[\mathbb{E}(Y \mid \Phi, D = 0) \mid D = 0] \\ &= (\mathbb{E}[\Phi \mid D = 1]'\gamma^{\nu}_{1} + \mathbb{E}\left[\varepsilon_1 \mid D = 1\right]) - (\mathbb{E}[\Phi \mid D = 0]'\gamma^{\nu}_{0} + \mathbb{E}\left[\varepsilon_0 \mid D = 0\right]), \end{aligned} \tag{8}$$

where $\mathbb{E}\left[\varepsilon_s \mid D = s\right] = 0$ because $\mathbb{E}\left[\varepsilon_s \mid \Phi, D = s\right] = 0$, so the expression reduces to $\Delta^{\nu}_{O} = \mathbb{E}[\Phi \mid D = 1]'\gamma^{\nu}_{1} - \mathbb{E}[\Phi \mid D = 0]'\gamma^{\nu}_{0}$. Thus, by adding and subtracting $\mathbb{E}[\Phi \mid D = 1]'\gamma^{\nu}_{0}$ we get

$$\Delta^{\nu}_{O} = \underbrace{\mathbb{E}[\Phi \mid D = 1]'\left(\gamma^{\nu}_{1} - \gamma^{\nu}_{0}\right)}_{\Delta^{\nu}_{S,OB}} + \underbrace{(\mathbb{E}[\Phi \mid D = 1] - \mathbb{E}[\Phi \mid D = 0])'\gamma^{\nu}_{0}}_{\Delta^{\nu}_{X,OB}}.$$